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Introductory Thoughts for Analysing the Landscape According to the Golden Ratio

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Abstract

Advancement of the cognitive approach is experienced not only in the field of psychology but interests numerous scientists in physical sciences as well. New relationships are recognised between distant science fields. This paper attempts to study the relationship between landscape ecology based landscape research and aesthetics – that is materialized primarily in arts. Mathematics gives the optics for the study and tourism benefits from it ultimately. Golden Ratio (marked by the Greek Φ) known and used already by ancient people occurs numerous times in composition (paintings, statues, pieces of music, literature works, buildings, etc.) regarded as aesthetic just like in the processes of Nature (e.g. growth, flowers, etc.). This lead to the idea to make mathematic calculations in the field of landscape aesthetics that is closely related to natural tourism. These calculations help to analyse the presence of the Golden Ratio in landscapes. Two methods or rather aesthetic indicators are proposed for these calculations. Exact values of these indicators make the landscapes comparable and thus rankable helping in this way tourists in choosing their destination.

Keywords: Aesthetics, Cognitive Approach, Golden Ratio, Landscape, Mathematics, Methodology, Tourism

Introduction

„We live in a universe of patterns where every night the stars move in circles across the sky. The seasons cycle at yearly intervals. No two snowflakes are ever exactly the same, but they all have sixfold symmetry. Tigers and zebras are covered in patterns of stripes; leopards and hyenas are covered in patterns of spots.... Human mind and culture have developed a formal system of thought for recognizing, classifying, and exploiting patterns. We call it mathematics. By using mathematics to organize and systematize our ideas about patterns, we have discovered a great secret: nature's patterns are not just there to be admired, they are vital clues to the rules that govern natural processes." This quotation is from the work of Stewart (1997) entitled „Nature's Numbers" and it summarises the main aim of this paper.

According to Penrose (1993) differential equations of physical sciences have to be considered seriously in modelling recognition (cognitive view). Its real pattern has to be looked for in the cognitive science – the primary aim of which is crossing the artificial boundaries of fragmented knowledge with maintaining scientific rationalism – in physics (Pléh, 1998).

Advancement of the cognitive view is significant not only in the field of psychology but it is a concern for many of those as well representing traditional physical sciences (Pléh, 2003; Kérdő, 2006). This process results in the acquisition or clearance of newer-and-newer interrelationships in relation of scientific fields considered distant from each other earlier.

This paper tries something similar as the relationship between landscape research based on landscape ecology and aesthetics – shaping primarily in arts – is viewed with mathematics providing the optics but tourism would be the end user.

Specifics of the Golden Ratio

Observation and systemizing of the phenomena of living Nature led to the formulation of Nature's laws in the language of mathematics over time. A

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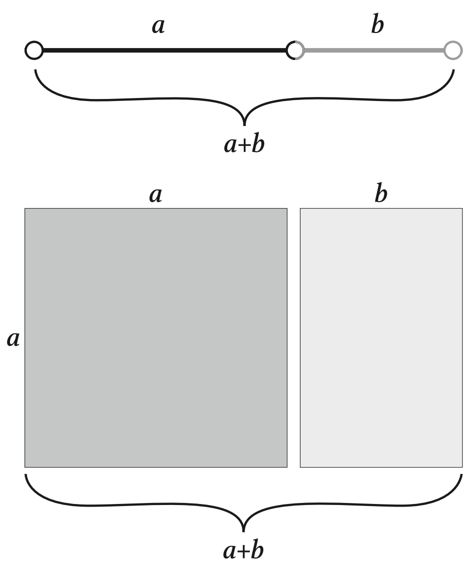
common characteristic of most of these laws is the significant role of the term ratio or its some sort of form played in them. However, these laws can be detected not only in Nature since for example the relation of sounds in music depends on the ratio of their number of oscillations and similarly ratios help the composers of buildings, statues and images in seizing reality and realizing their artistic efforts (Livio, 2002; Hemenway, 2009) [1].

Due to its frequent appearance in both the forms of living Nature and works of art special attention has been paid for a special symmetry since the antiquity. Until the exact reasons and conditions of its formation were unclear its appearance was associated with mystic meanings. This mysteriousness is reflected in its name: *Golden Ratio* or *Sectio Aurea* in Latin, even the names sacral or saint geometry may also occur [2].

It was called divine number in the antiquity as people regarded it not only as a mathematic fact but as the evidence for the presence of God on Earth and the sign of creation. This understanding is reflected in the book entitled *De Divina Propotione* (*The Divine Ratio*) of Pacioli (1445-1514) the Italian mathematician and Franciscian friar written in 1509.

The Golden Ratio is a proportion that can be detected frequently in the reality surrounding us creating a natural balance between symmetry and asymmetry. In order to understand the listed examples we have to overview the mathematic characteristics of Golden Ratio first.

Mathematic definition of Golden Ratio: two parts (a and b, a>b) according to the Golden Ratio are in proportion to each other if the whole (a+b) is in proportion to the greater part (a) exactly as the greater part (a) to the smaller (b):



$$\frac{a+b}{a} = \frac{a}{b}$$

Illustration 1 and 2. *Source: [3]*

I.e. half the length of the greater part equals the geometrical centre of the length of the sum and the smaller part:

$$a^2 = (a+b) \times b$$

In other words the greater part is in proportion to the smaller part as the smaller part to the difference of the two parts. Pythagoreans regarded the rectangle the most aesthetic the sides of which all meet the Golden Ratio.

$$\frac{a}{b} = \frac{b}{a-b}$$

that is

$$b^2 = a \times (a-b)$$

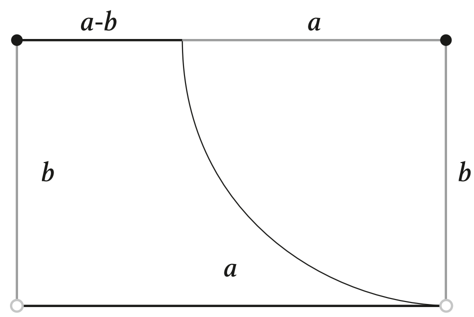


Illustration 3. *Source: [4]*

Golden Ratio is marked by the Greek letter 'Φ'. It comes from the name of Pheidias a Greek sculptor and architect who used the Golden Ratio often. The constant in question is frequently marked by the Greek letter 'τ' as well, it can be seen mainly in the works of Penrose (1993).

Φ is an irrational constant, i.e. a real number that cannot be stated as the quotient of two whole numbers, i.e. it is not a recurring decimal. Its approximate value: $\Phi \approx 1.618$... This number is sometimes indicated as $\varphi \approx 1.618$ but in this case it marks $\Phi \approx 0.618$ a b/a [1].

From the above definition the ratio of the greater part (a) and the smaller (b) can be calculated and in this way the Φ value can be obtained for which

$$a = \Phi \times b \quad \text{i.e.} \quad \Phi = \frac{a}{b} \qquad \text{is true.}$$

According to the definition dividing both the numerator and the denominator by 'b':

$$\frac{\mathrm{a}}{\mathrm{b}} = \frac{\frac{\mathrm{a}}{\mathrm{b}}+1}{\frac{\mathrm{a}}{\mathrm{b}}}$$

Substituting $\Phi = \frac{a}{b}$ in this:

$$\Phi = \frac{\Phi + 1}{\Phi}$$

multiplying this by Φ then arranging it to 0 the following quadratic equation is obtained:

$$\Phi^2 - \Phi - 1 = 0$$

This equation can be solved by the following solution formula:

$$\Phi = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times (-1)}}{2 \times 1} = \frac{1 \pm \sqrt{5}}{2}$$

Negative root of the equation (≈-0.618) is not solution of the problem due to the nature of the problem thus:

$$\frac{a}{b} = \Phi = \frac{1 + \sqrt{5}}{2} \approx 1.618\,033\,988\,749\,89 \ldots$$

or

$$\frac{b}{a} = \Phi = \frac{\sqrt{5} - 1}{2} \approx 0.618$$

Its graphical construction can be made on the basis of several mathematic theses – e.g. 47th thesis of Euclid in book I of its work entitled *Elements* (*Stoikheia*) and 11th thesis of book II [5]; the perpendicular sides thesis; thesis of tangent and secant sections drawn to the circle from an outer point; etc.

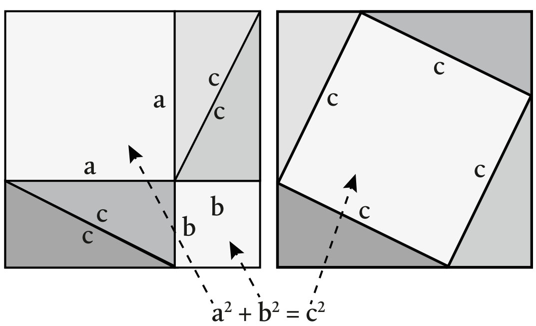


Figure 1. Verification of the Pythagoras thesis graphically
Square of the longest side (hypotenuse) of any rectangle triangle equals the square sum of the other two sides (perpendicular sides). In other words: if a triangle is right angled the area of the square drawn on its longest side equals the sum of the area of the two squares drawn on the two other sides. With the usual symbols ('c' is the hypotenuse): $a^2 + b^2 = c^2$.
Source: [6]

The most widely known among them maybe the Pythagoras thesis (Figure 1) that is accepted as an axiom in Euclid's geometry. With the Cibulka type repeated application of this Φ can be drawn graphically (Czibulka, 2007).

Fibonacci Numbers

In algebra relations the Golden Ratio is frequently mentioned together with the so called *Fibonacci Numbers* that were presented first by Pisano (1170-1250) in his *Liber Abaci* (Book of Calculation) in 1202 on the example of the growth of an imaginary rabbit family. Kepler (1571-1630) in his book entitled *The Six-Corenered Snowflake* published in 1611 re-discovered this recursive sequence and associated it with various natural phenomena. Its current name was given by Lucas (1842-1891) [7].

Relationship between the Golden Ratio and the Fibonacci Sequence is that the limit of the sequence given by the quotient of the subsequent Fibonacci Numbers (1/1, 2/1, 3/2, 5/3, …) is exactly the Golden Ratio, i.e. Φ. The first two members of the Fibonacci Numbers are 0 and 1. Then each subsequent numbers is the sum of the previous two (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89…).

In mathematical terms the sequence of Fibonacci Numbers is defined [7]:

$$a_1 = 1, a_2 = 1 \quad \text{and} \quad a_{n+1} = a_n + a_{n-1}, \quad \textbf{ha } n > 2$$

Reducing both sides by a_n:
The obtained equation would be the same as

$$\frac{a_{n+1}}{a_n} = 1 + \frac{a_{n-1}}{a_n}$$

the Golden Ratio equation if the quotient of the subsequent elements of the sequence of Fibonacci Numbers was the same, i.e. the elements would make a geometric sequence. This is not true, however, as the quotient of the subsequent elements is not constant. This is especially apparent in the case of low numbers (Figure 2). Increasing the number of elements, however, this quotient approaches a constant number, the Φ.

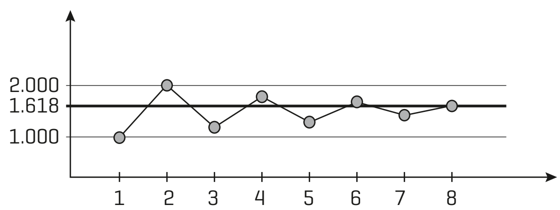


Figure 2. Quotients of the Fibonacci Numbers approach the constant Φ from two sides
Source: [8]

Examples for the occurrence of the Golden Ratio

After understanding the mathematical background of the Golden Ratio let us see some examples for it. It has been proved that as early as in the ancient Egypt Golden Ratio was understood and applied as it can be detected in the great pyramid in Gíza that was built around 2600 BC. Half of the base edge of the pyramid (186.42 m on average) and the height of its side sheets (around 115.18 m) are in proportion to each other as the Golden Ratio with 0.03% of deviation which can be regarded as within the limit of error [9].

Ancient Greek people known and praised this ratio. It was considered by Pythagoras, Theodorus and Euclid as well in geometric calculations. The ancient symbol, the pentagram often occurs in relation to the Golden Ratio and this has been interpreted by many in many different ways. If a standard pentagon is drawn, its sides can be divided automatically by the Golden Ratio and the ratio of the line sections is always Φ. This is why this shape became the symbol of the Golden Ratio and thus that of perfection.

Pythagoreans called the pentagram (peak downward) Hygieia, Greek name for the god of health. It was the symbol of Venus in Rome, the goddess of love and beauty (i.e. aesthetics), however, it also symbolizes Jesus (morning star, star of Betlehem). In the Middle Ages the five principle elements (water, earth, fire, air, soul) were marked by it. It was also regarded as the key for sciences, and for secrets. It is the most powerful symbol according to Paracelsus (Szemadám, et al., 2000).

Golden Ratio is the basic organising structure for example in the sonatas of Mozart; V. symphony of Beethoven; compositions of Bartók, Kodály, Debussy and Schubert; just like in the *Divine Comedy* of Dante and the poem of Lajos Kassák entitled: *The Horse Dies – the Birds Fly Out*. In the arrangement of the most known compositions – e.g. those of Leonardo da Vinci (Figure 3) or Michelangelo – Golden Ratio is applied similarly to the parameters of the Hungarian Saint Crown (Ferencz, 2002). This approach is also widespread in the works of contemporary painters, to mention only the MADI movement and the works of its significant Hungarian representative, János Saxon-Szász (Perneczky, 2002) [10].

The famous Vitruvius Man (Figure 4) is also the work of Leonardo da Vinci in which he describes the accurate proportions of the bone structure of a man. He was the first to suggest that the human body is composed of "building stones" the proportion of which is always Φ (Frissell, 2009).

Later Zeising (1810-1876) in his work entitled *Auf experimentalen Asthetik* (*Experimental Aesthetics*) writes about his measures performed on the

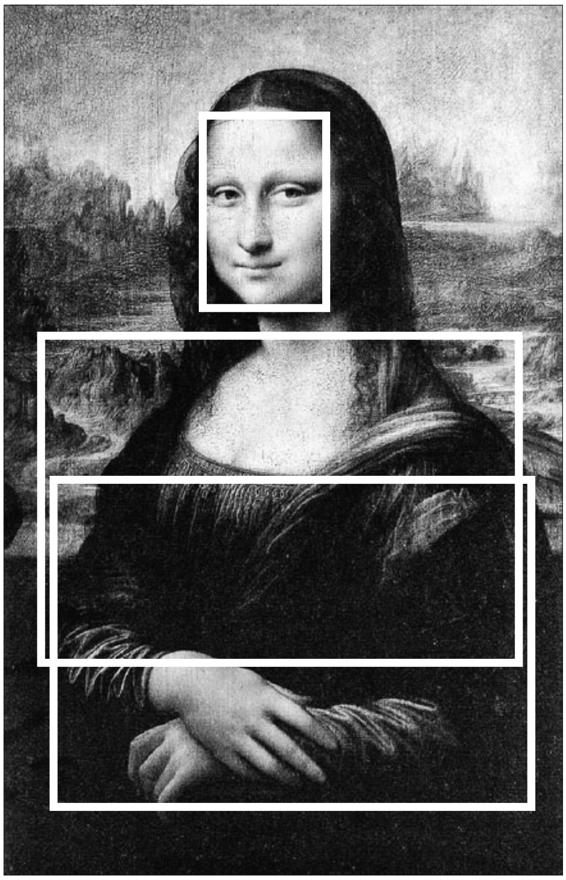


Figure 3. Occurrence of the Golden Ratio in the painting of Leonardo da Vinci entitled: Mona Lisa
Source: [11]

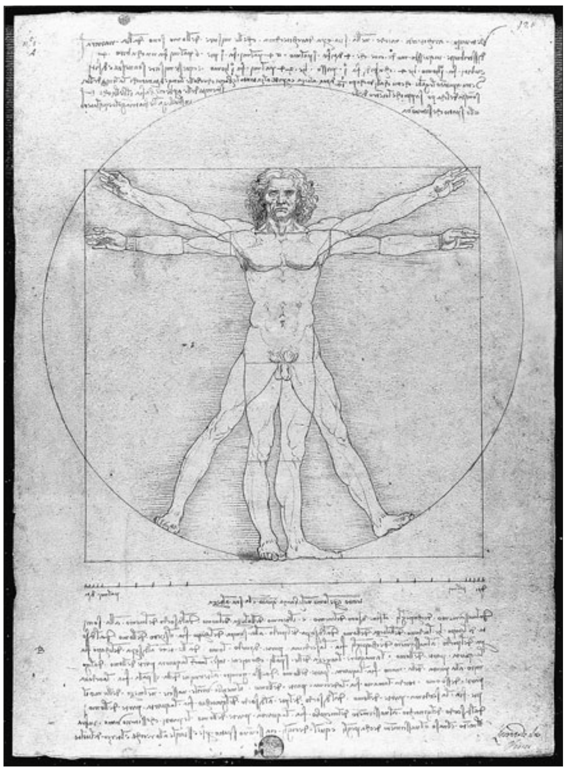


Figure 4. The Vitruvius Man of Leonardo da Vinci
Source: [12]

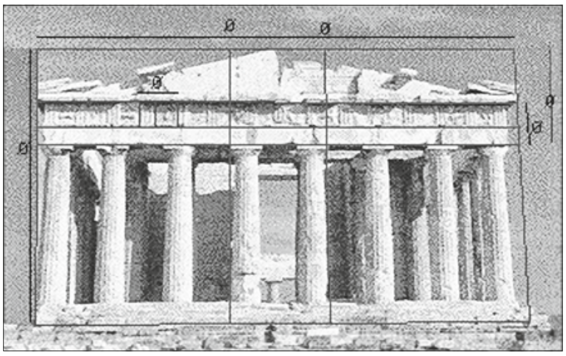


Figure 5. Golden Ratio in the building of the Parthenon in Athens
Source: [13]

human body. He stated that the jointing points of the trunk and main parts of a body have a proportion equalling the Golden Ratio. When the height of a body is 1000 then the lower part from the navel is 618, the upper part from the navel is 382 and the length of the head is 146. These all have the proportion of the Golden Ratio.

Zeising tried to show the Golden Ratio in the dimensions of the whole and parts of the most renowned buildings of antiquity and the Middle Ages. A fine example for this is the Parthenon in Athens (Figure 5) the statics of which is given by two squares that can be drawn in it and its dynamics is given by the Golden Ratio [4].

Based on the examples from arts it can be stated that Golden Ratio is regarded to be a perfect proportion having aesthetic value. Its perfectness is given by not regular symmetry but the harmonic balance of symmetry and asymmetry and that is why people regard it aesthetic.

This is supported by the fact that perfect symmetry seems to be less natural for the human observation. Far more exciting if it is "damaged" somehow (Sailer, 1994; Mainzer, 1996) because that is the beginning of something else (Kérdő, 2006). This is the base of growth in Nature, i.e. in the physical space.

The Φ proportion is present in almost everywhere in Nature, it is its elementary constituent. Spatial specifics of plants, animals and even men (see above the analyses of Leonardo da Vinci and Zeising) show surprisingly accurate the Φ to 1 proportion. In other words, Nature copies this proportion again and again and this cannot be pure accident. This is why people in ancient times regarded Φ as the number determined by the creator of the universe, a divine number [2].

Most widely known examples from Nature: proportion of one spiral of the nautilus or nautili house (Figure 6) to the other is Φ, i.e. AC:DB=FG:EG (Figure 7). Proportion of female bees to male bees in a beehive or the proportion of the parts of an insect (head-thorax-abdomen) to each other is also Φ. Similar structure can be detected in a spider web as well.

Lamellae of the pine-cone and that of the pine apple or the seeds in the sunflower head are placed

Figure 6. Cross section of the nautili house
Source: [14]

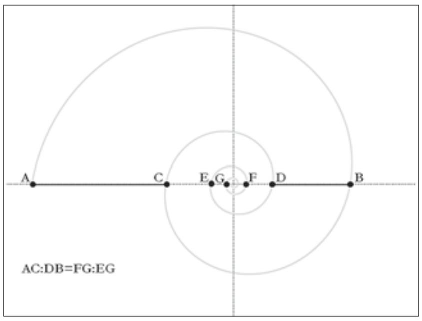


Figure 7. The logarithmic scale and the Golden Ratio
Source: [15]

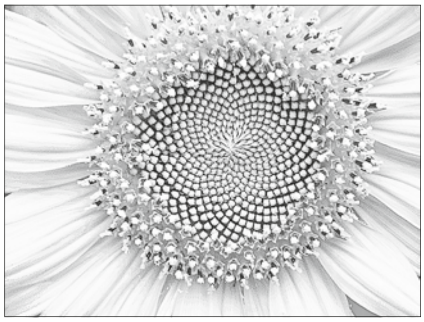


Figure 8. Double spiral arrangement of the seeds in the sunflower head
Source: [16]

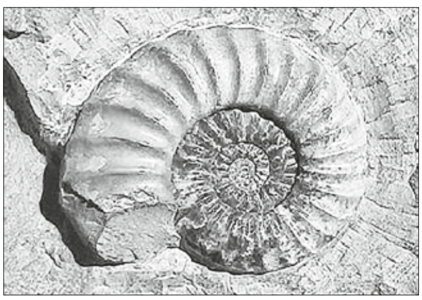


Figure 9. Lithified skeleton of an ammonite
Source: [17]

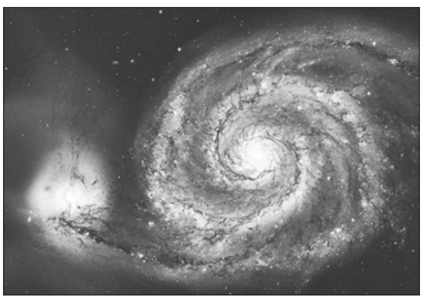


Figure 10. Spirals of the Messier 51 galaxy
Source: [18]

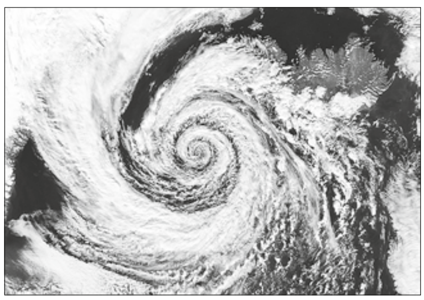


Figure 11. Cyclone above Iceland
Source: [19

Figure 12. New shoot of common male fern
Source: [20]

along two opposite logarithmic spirals (Figure 8) where the proportion between the diameter of two neighbouring rows is Φ. It is also frequent that leaves (phyllotaxis), buds and petals are ordered not symmetrically. Such order composes the onion head of the onion leaves, the cabbage head of the cabbage leaves furthermore the raspberries, roses of the cauliflower and the thorns of certain cactuses are also arranged in this order (Church, 1904).

Similar to the house of the nautili the lithified skeleton of ammonites that lived dominantly in the Palaeozoic and Mesozoic (Figure 9) also shows the characteristic logarithmic spiral structure just like most of the galaxies that have spiral structure (Figure 10). Typical cyclones in the Atmosphere also take this form (Figure 11). Flying trajectory of falcons also forms a logarithmic spiral when it approaches its prey watching it in this way always from the same angle.

In spite of listing endless numbers of examples I would like to draw attention to the similarity that can be detected in certain phases of growth of living beings both in the world of plants and animals and thus in the case of human beings as well (Figures 12 and 13). Based on these images this spiral form seems to be dominant over the development of life as well and deviation from it can be regarded as exception (Mainzer, 1996). But what is this splendid spiral, how does it form and what is its mathematic base?

The Golden Spiral

Maybe it is not surprising that in this respect again the Golden Ratio and the Fibonacci Numbers and the Fibonacci square that can be derived from it play the main role (Figure 14). Significance of the Fibonacci square – its sides are Fibonacci Numbers – is that it gives a very good approach to the construction of the Golden Spiral that is increasing by Φ (Figure 15). Therefore the spiral is often called Fibonacci Spiral (Mainzer, 1996; Hemenway, 2009) [1].

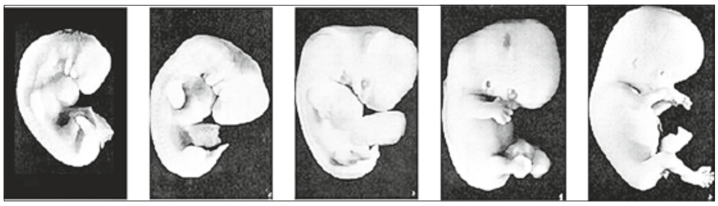


Figure 13. Phases of the development of a human embryo
Source: [21]

The Golden Spiral was described first by Descartes (1596-1650) then later it was studied by Bernoulli (1654-1705) who was so marvelled by the unique mathematic characteristics of the spiral that he called it the "Miracle Spiral" (*Spira Mirabilis* in Latin). Distance between the turns of the spiral increases according to a geometric sequence but its shape remains unchanged despite any further twist. Probably this is the reason why several natural forms associated with growth have this logarithmic spiral shape as shown in figures 6-11 [1] [24].

Logarithmic spirals are similar and congruent (fit with) to themselves for all similarity transformations. In other words their enlargement-re-

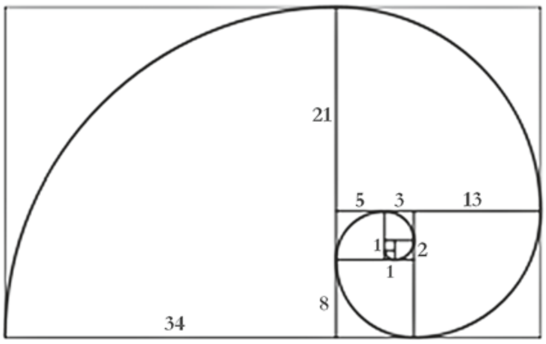


Figure 14. Fibonacci Square and Spiral
Source: [22]

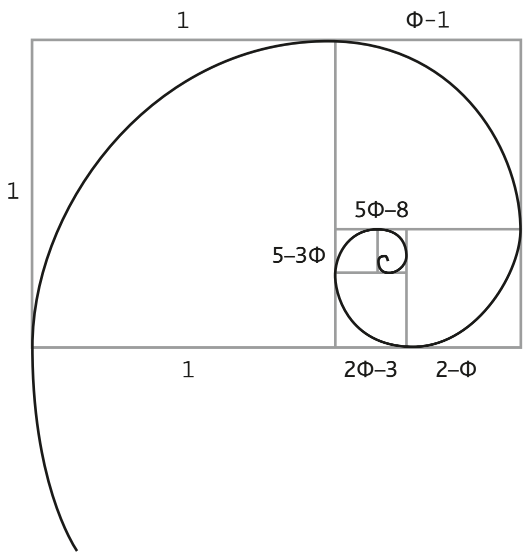


Figure 15. Construction of the Golden Ratio
Source: [23]

duction results in the same form as their rotation around the pole. Their enlargement by the factor $b^{2\pi}$ yields the original curve. They are also congruent to their evolutes and evolvents. Within the group of spiral plane-curves the Fibonacci Spiral is a logarithmic spiral that increases by Φ over a quarter turn, i.e. the equation of the spiral is: $c \times \Phi^{2/\pi}$ [24].

Polar equation of the Golden Spiral is the same as that of other logarithmic spirals, however, with a specific *b* value that is the extension factor of the spiral and it is dependent on Φ covered by every quarter of a circle:

$$r = ae^{b\theta}$$

where ***e*** is the base of natural logarithms while ***a*** is a optional positive real constant.

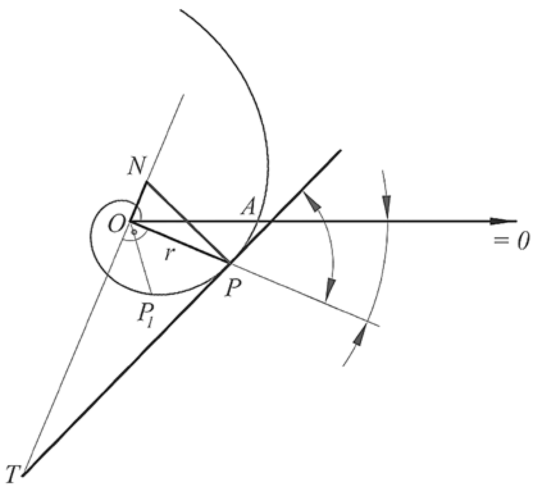


Illustration 4.
Source: [24]

Or in a different form and this is where the „logarithmic" name comes from:

$$\theta = \frac{1}{b}\ln\left(\frac{r}{a}\right)$$

thus, a ***b***:

$$b = \frac{\ln \Phi}{\theta_{right}}$$

Value of *b* depends on whether the right angle is 90 degrees or π/2 radian and since the angle can be orientated in both directions it is better to apply the absolute value of *b* in the equation (*b* can be the additive inverse of this value as well).

At θ degree:

$$|b| = \frac{\ln \Phi}{90} = 0.0053468$$

At θ radian:

$$|b| = \frac{\ln \Phi}{\pi/2} = 0.306349$$

Equation system of the curve in parametric form:

$$x(t) = r\cos(t) = ae^{bt}\cos(t)$$

$$y(t) = r\sin(t) = ae^{bt}\sin(t)$$

where *a* and *b* are real numbers [24].

Curved lines – thus spirals as well – are generally associated with some sort of a movement, rhythm. Human eye follows these easily. Curved lines compared with straight ones express greater dynamism this is why they are dominant elements of any composition, picture, statue or any other composition. As they are more frequent in Nature, curved lines are felt much more natural and therefore aesthetic for humans than the more regular, i.e. symmetric straight lines [24]. Just think of the sight of a naturally meandering river and an artificial canal.

In summary the significance of the Golden Ratio is that the popularity of forms holding the Golden Ratio has been unbroken since the antiquity no matter whether they are statues, buildings, paintings or other compositions, they all have great aesthetic value. Golden Ratio is therefore is a proportion that expresses what is thought to be aesthetic regarding spectacle. Taking a comparison, music is the harmony of sounds while harmony of the Golden Ratio is spectacle. And this leads us to the issue of landscape view.

Golden Ratio studies in landscapes

In the course of landscape aesthetic studies the question often arises, how aesthetic a landscape is and – eliminate the apparent large grade of subjectivity of aesthetic quality – how its exact measurement could be made possible. Obtained factors would enable the comparison of the landscapes. This is targeted in the followings.

If we continue on the logic supported by the above examples and we accept that Nature copies Φ and the landscape is the integrate part of Nature then it seems reasonable to study landscapes according to the rule of the Golden Ratio and to investigate how Golden Ratio that expresses the aesthetic harmony of symmetry and asymmetry is reflected in the landscapes.

There are numerous variations to the question of how and for what landscape elements the Golden Ratio can be investigated in the given landscape – or considering the view of the paper in the given small landscape unit. To study and calculate all of these for all the 230 small landscape units of Hungary would be a significant research task.

Preparing a GR test network

In the first round of investigation let's start from the analogy that Leonardo da Vinci called bones the "building stones" of the human body and rules of the Golden Ratio seem to be in the proportion of these bones. Considering this it can be stated that the "building stones" of landscapes are the elementary landscape parts that are termed ecotopes in international terminology and landscape cells in Hungarian terminology (Mezősi, 1991; Csorba, 1996; Lóczy, 2002; Kertész, 2003; Kerényi, 2007).

It is sensible to start landscape investigations from the Golden Ratio point of view at ecotope level. The issue can be investigated from several directions. One possible investigation way is to study the ecotopes located in the Golden Ratio points (**GR points**) of the given small landscape unit and they can be grouped according to the hierarchy of GR points (primary, secondary, tertiary). These can be weighted if for example the area of the ecotopes located in the Golden Ratio points is considered. The obtained indicator – we can call it **GR index** – makes two or more small landscape units comparable mathematically.

Principle and problems of constructing test square network including the GR points are presented by Figures 16-20.

Taking the calculations of Zeising on the human body (he stated that if the height of the body is 1000 then the lower part from the navel is 618 and the upper part from the navel is 382 units) it can be applied to the kilometre that is composed of 1000 metres. As the kilometre network used in the case of maps consists of 1km x 1km square network it can be overlapped completely by a **GR test square net** (GR net in short) that is fit onto the rectangular intersections of the kilometre network (Figure 16).

Based on this, one plane of Golden Ratio can be drawn at 618 metres right from the focus on the X-axis orientated West-East of the GR network. Measuring the same on the South-North orientated Y-axis the first intersection of the Golden Ratio (A) is obtained. In case this measurement is performed from every corner that can be regarded as focus the rest of the primary GR points (A'; A"; A"') and all of the golden ratio planes of the GR test net are obtained and these divide the test net symmetrically by a standard cross. Secondary GR points are located within this cross while tertiary GR points are outside the cross but they are still located along the planes of the Golden Ratio.

The two squares marked by the (A), i.e. the WE (X-axis) –SN (Y-axis) orientated Golden Ratio plane within the test square are coloured green. The two squares are asymmetric to each other regarding both size and place within the net. This

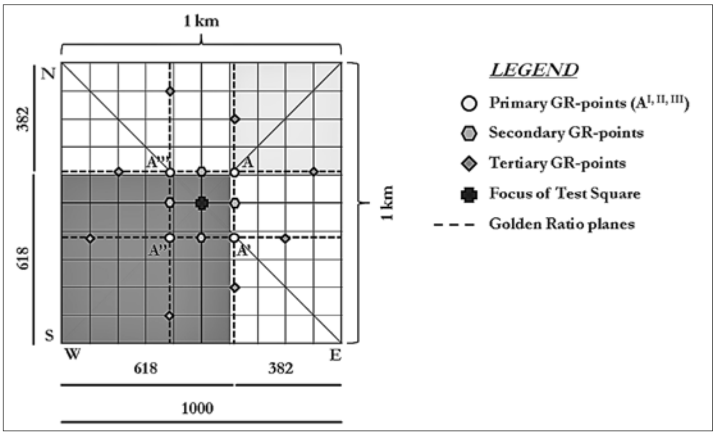


Figure 16. Theoretical outline of the GR test net following the rules of the Golden Ratio and the Fibonacci Numbers

Note: this GR test square is the same as the graphical expression of the Pythagoras thesis shown in Figure 1.
Source: own construction

asymmetry is the result of the division according to the Golden Ratio. For easier understanding only this WE–SN orientated (A) intersection plane and the associated intersection points are considered in the followings, however, the analysis will only be complete when it is performed for the rest of the Golden Ratio planes (A'; A"; A"').

As the factor of the Golden Ratio, the Φ is an irrational number, i.e. infinite non recurring decimal it is difficult to calculate with it in the decimal system. This is why the not too round number of 618 was obtained for 1000. This number is not easy to measure in the field, even using a GPS the measurement can be inaccurate as their accuracy is only within 5-10 metres and this can be considered too big for certain ecotopes to decide whether they are within the Golden Ratio plane or not.

An even greater difficulty is presented by that neither the 10 nor the 1000 are Fibonacci Numbers but sticking to the 10x10 test square net is important in order to stay comparable to maps. Therefore the construction of a simplified test net that can be used in the field safely but still follows the rule of Golden Ratio considering the division of the test area is justified (Figures 17/a and 17/b).

The significant specific of this Figure is that it operates according to the rule of Fibonacci Numbers but with whole and not irrational numbers resulting in a test net easy to construct and better to understand.

Starting from the 10x10 net we have to pick those whole numbers in the interval from 1 to 10 the quotient of which approaches best the Golden Ratio. Good news is that there is only one pair of numbers in this interval. This can be given according to the Fibonacci principle as follows:

$$2 + 3 = 5 \textit{ and from this } 5 \times 2 = 10$$

According to the rule of the Golden Ratio the smaller part is proportional to the greater one as the greater part is proportional to the whole:

$$2/3 = 0.66667 \textit{ and } 3/5 = 0.6$$

It has to be noted that the two results are not exactly the same but still this pair of numbers approach the best the Golden Ratio factor (Φ≈0,618) in the interval from 1 to 10. Based on these the following Figure is obtained (Figures 17/a and 17/b).

If the obtained primary GR points are studied then based on Figure 17/a:

$$3/(3+4) = 0.4285...$$

Value of the quotient will be even further from Φ if section 2 is taken first during the construction (Figure 17/b). In this case:

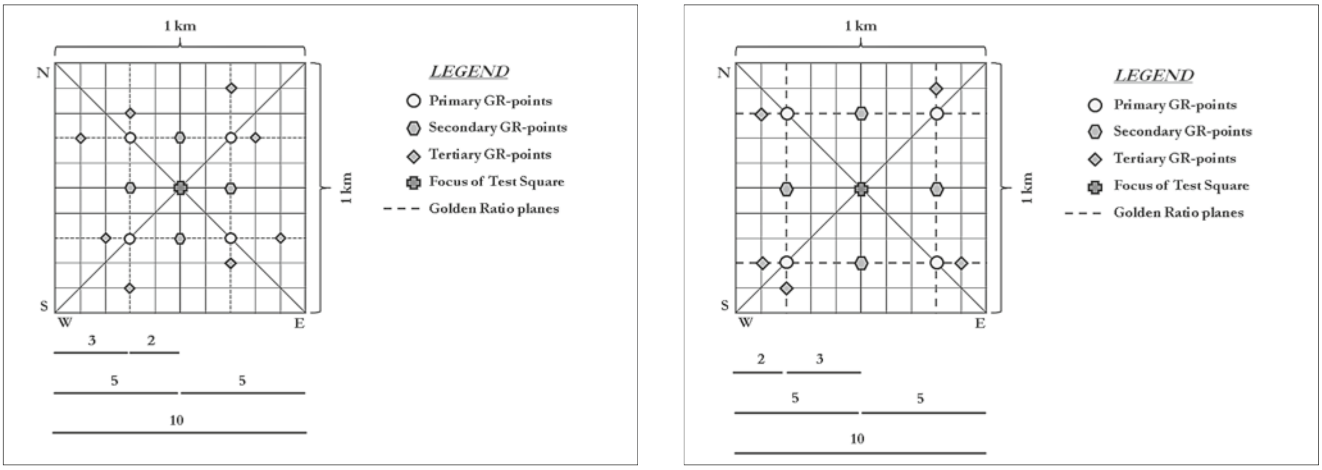
$$2/(2+6) = 0.25$$

It can be stated therefore that the Fibonacci Numbers combination of 2, 3 and 5 related to the 10×10 test net yields no distribution according to the Golden Ratio for any of the intersection points.

Let's see the problem constructing the Golden Ratio planes with the same number pair in a 5×5 square (Figure 18).

Please note that the test net constructed in this way is greatly similar to the one in Figure 16. The difference is only that this version is for whole numbers but the pattern is almost identical. Difference between the patterns of the two figures is exactly the amount of mathematic difference among 0.6667, 0.6 and 0.618, however, this difference remains within the theoretical limits of the sequence of Fibonacci Numbers.

Thus all we have to do is to copy the structure onto the other 5×5 parts of the test square. The method is the same to extend the pattern to neighbouring test squares, i.e. translating the structure along the (blue coloured) diagonal lines of the test squares we obtain a template (Figure 19).



Figures 17/a and 17/b. Simplified versions of the Golden Ratio and Fibonacci Numbers based test net for whole numbers
Source: own construction

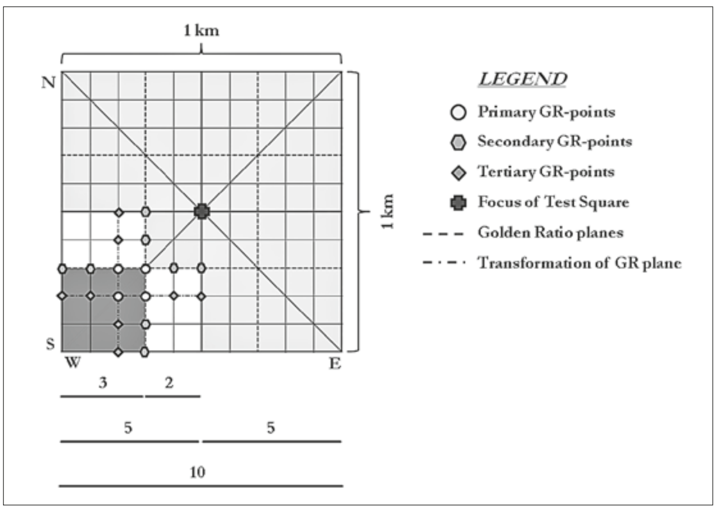


Figure 18. Simplified versions of the Golden Ratio and Fibonacci Numbers based test net for whole numbers in a 5x5 square
Source: own construction

Copying the pattern four times the 4 primary GR points of the 10×10 test net is obtained and they mark the square within the net that is called **Golden Square** due to its significant role in the Golden Ratio. This 5×5 square covers ¼ of the area of the test net but it is located in the part of the net that divides the area of the test net according to the Golden Ratio in the form of 3+5+2.

As the Golden Square is situated in the same part of all the test squares Golden Ratio is the same in it therefore it can be stated that if ecotopes are analysed in this Golden Square located between 300 and 800 metres from the points of adjusting taken as focuses (O) in all of the test squares then ecotopes make studied landscapes comparable regarding Golden Ratio.

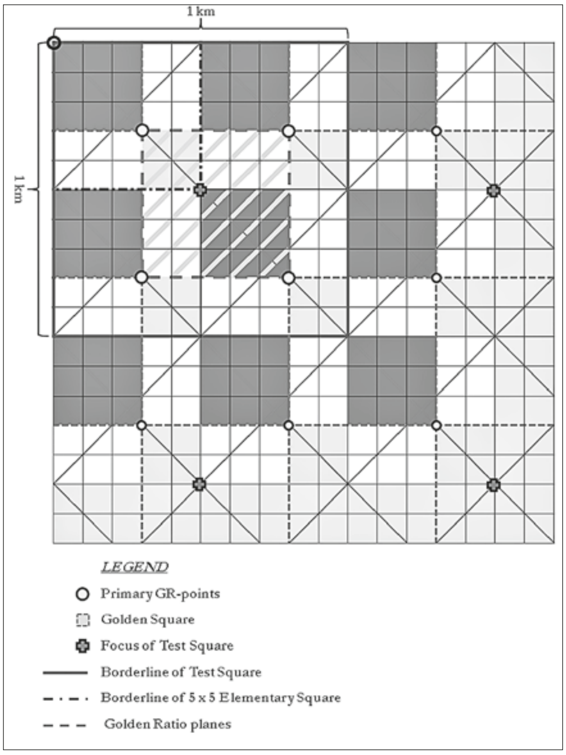


Figure 19. Test template based on the Golden Ratio and the Fibonacci Numbers
Source: own construction

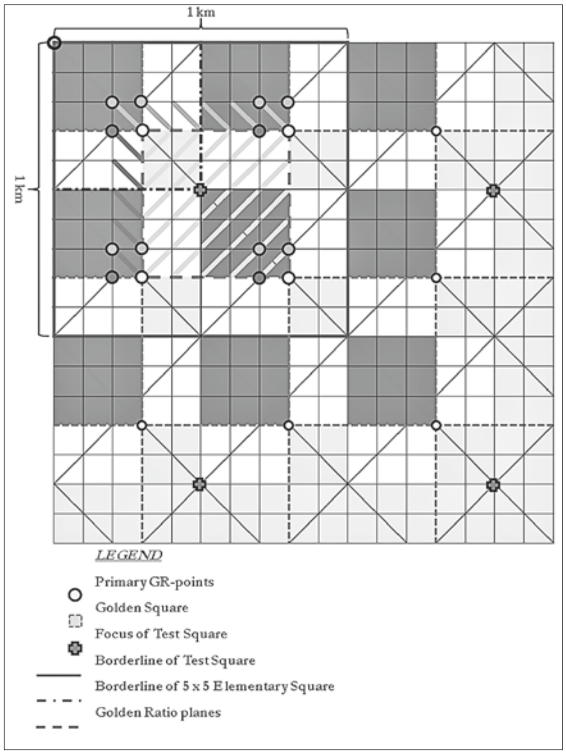


Figure 20. Golden Square and its transversions according to the primary GR points within the test template
Source: own construction

If primary GR points are marked from all corners of the test square then the Golden Square of Figure 19 is extended by a belt of 100x100 metres towards SW (Figure 20). In this way this square puts the points located in this valuable part regarding Golden Ratio between 200 and 800 metres in the test net.

Golden Ratio analyses using the Golden Spiral

In the second phase of analysis we can calculate whether ecotopes composing the small landscape units or even the landscapes forming the country follow logarithmic spiral based on the Golden Ratio or not. An even their difference from it can be given mathematically. If the ideal case is the pattern according to the Golden Spiral and take it as 100% then the difference from it can be given by a simple mathematic operation. The index obtained in this way could be called GR_S index – as originated from the spiral – based on which our landscapes could be compared easily and an aesthetic order considered as exact could be set.

Although photo analysing softwares applying the Golden Ratio are already exist [25]; [26] but these are not free today that also justifies the need to construct a standard vector spiral with unit size using the software ArcView 3.2 that operates similar to the linear scale of maps. And this would be adjustable to maps of any scale due to the congruent characteristics of the spiral.

Starting point of the spiral measure can be adjusted to any points of the study area, however, in order to remain comparable it is sensible to choose a fix point that can be defined accurately in the case of every landscape. Therefore we recommend one of the primary GR points defined by the above method or the geometric centre of the landscape or the focus of the GR test net closest to that. It is a free choice, however, it is important to give accurately and united the method of selecting the adjustment point for every landscape.

Conclusion

The presented two methods are suitable separately for finding harmony so important considering aesthetics based on the balance of symmetry and asymmetry in the given landscape or in the image depicting it by mathematic procedures. GR indexes and GR_S indexes calculated in this way can be used for the comparison and ranking of our landscapes.

Such classification of landscapes could yield information useful for visitors as well because exact parameters help them to decide which landscapes to visit during their journey if they wish to see something really aesthetic, if they are longing for the harmony provided by the landscape view.

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