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FROM THE WEBER PROBLEM TO A "TOPODYNAMIC" APPROACH TO LOCATIONAL SYSTEMS[1]

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Abstract

In any given space, a sequence of interdependent Weber problems of a certain type leads to a pattern of locations which can be mathematically characterized. Conversely, the observed evolution of a given locational system corresponds to certain characteristics of an analogous weberian locational system. Determining such characteristics leads to simulating and forecasting the evolution of the observed locational system. A model corresponding to such a "topodynamic" approach is presented and an application is made. The model integrates three different effects: an interdependency effect which determines the polarization level; an "attraction-repulsion" effect which determines the center-periphery equilibrium; finally, a distance deterrence effect which determines the diffusion process.

Introdaction

The so-called "Weber problem" traditionally has been defined as a total transportation cost minimization problem with respect to a space described as a "uniform plain". In Tellier (1972) and (1985), it was suggested that the Weber problem is much more general than its original formulation and that it can refer to any location problem that involves the optimization of a continuous differentiable location function L = L (g_l,..., g_i,...,g_n) defined in terms of Euclidean distances g_i to reference points 1,..., i, ..., n, and characterized by the fact that $\partial L / \partial g_i$ is the same for every value of g_i and for every direction in a two-dimensional space. The functional form of L is the following:

$$L = \sum_{i=1}^{n} w_i g_i$$

where

$$g_i(x) = |x - x_i|$$

x_i: the location vector of the i-th "facility" (or reference point).

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In the traditional formulation of the Weber problem, L was a total transportation cost function that had to be minimized and w_i (that is $\partial L/\partial g_i$) was defined as a non-negative value equal to a transportation rate times a corresponding quantity of conveyed goods.

More generally, Weber problems can involve both attraction points (and forces) and repulsion points (and forces). Except for Wesolowski (1977) who referred to rectangular distances, for Tellier and Ceccaldi (1983), for Tellier and Polanski (1989) and for Drezner and Wesolowsky (1989), the case of repulsive forces has generally been ignored in the literature. An attraction point is such that its corresponding force $\partial L/\partial g_i$ is negative when L is to be maximized and positive when L is to be minimized. By contrast, a repulsion point is such that its corresponding force $\partial L/\partial g_i$ is positive when L is to be maximized and negative when L is to be minimized. A repulsion point represents an activity which the decision maker wants to be as far away as possible (for example, it may be a noisy airport, a nuclear plant or a dump).

This papers deals primarily with the "dynamic" case of successive interdependent triangle Weber problems. However, fundamentally, its results are based on the procedures[2] and the static analysis of the Weber problem presented in Tellier and Polanski (1989). The dynamic aspect of the question has been examined by Tellier and Ceccaldi (1983), Tellier, Ceccaldi and Tessier (1984) and Tellier (1987).

It is here suggested that resorting to a series of randomly determined and interdependent triangle Weber problems in a two-dimensional bounded space allows to simulate the observed evolution of locational patterns:

- when new activities are allocated among existing locations;
- when new activities are allocated among both existing and new locations;
- when available data are restricted to the geometrical coordinates of locations and to the distribution of activities among these locations;
- when the evolution of locational patterns is influenced both by distances between locations and by the geometry of the considered space (for example, by the distinction between center and periphery, or by existing borders or frontiers). It must be stressed that the presented approach is particularly worthwile in the case of processes involving the appearance of new locations, limited data and significant geometric effects. It is so as it distinguishes three essential aspects of locational evolutions: the evolution of the degree of polarization, the evolution of center-periphery equilibrium and the evolution of diffusion phenomena. These aspects are here treated by means of the Cameroon urbanization example (the proposed approach being useful in studying any locational evolution of human activities, illnesses or animal and vegetal species).

The Interdependency Effect

Location processes can be generated from Weber problems provided that a certain interdependency exists between successive problems. Such an interdependency here takes the form of a certain probability of selecting some of the three reference points of the successive triangle Weber problems among the points which have been chosen as optimal locations in the context of previous Weber problems.

[2] Weber problems involving only attractive forces are resolved by means of the Tellier (1972) solution and those involving both attractive and repulsive forces are resolved by means of the procedure presented in Tellier (1985) and in Tellier and Polanski (1989). The classical Kuhn and Kuenne (1962) method has also been used for testing.

Let us imagine a location process composed of a series of n successive Weber triangle problems involving location forces selected at random, the forces in absolute value being independently distributed with an identical uniform distribution U(0, k). Let us moreover imagine that the with Weber triangle problem involves reference points chosen partly among the points that have been found to be optimal locations in the (i -1) previous iterations and partly among all the points of the defined space, the points being independently distributed with an identical uniform distribution U(0, m) x U(0, n).

Such a process can be characterized by the following elements:

1- the degree of interdependency i of iterations (probability that an optimal location of a previous iteration be taken as a reference point for a given iteration);

2- the proportion a of attraction points among the reference points considered;

3- the supposed friction of the space p (probability that certain randomly selected location problem be rejected because its reference points are too distant from each other);

4- the number n of iterations realized during a simulation.

The degree of interdependency i plays a major role. According to our results in the context both of a theoretical rectangular space and of the Cameroonian territory, this characteristic determines to a very large extent the level of polarization[3] of locations. Comparing locational patterns in Figure 1 leads to such an intuitive conclusion that a more mathematical analysis only confirms.

The `Attraction-Repulsion` Effect

Despite the fact that, in the Weber triangle case, the level of polarization is not really influenced by the proportion of repulsion and attraction points in the system, this proportion has a major impact on the geometric form of polarization, as it can be observed in the Cameroon case (Figure 2). The higher the proportion of repulsion points, the more activities are located at the periphery. Conversely, the higher the proportion of attraction
points, the more activities tend to locate close to center.

The "attraction-repulsion effect" allows to take into account many locational phenomena which are linked to the existence of borders and territorial limits. It must be stressed that neither the interdependency effect nor the distance deterrence effect adequately deal with these phenomena. Every time a center-periphery disequilibrium exists, the attraction-repulsion effect may play a major role in simulating the observed evolution.

The Distance Deterrence Effect

The third effect included in the model concerns the deterrence effect of distance on location decisions. In the Weber problem context, this effect leads to rejecting with a certain probability location problems involving too great distances. The acceptance probability function may take various forms. In the Cameroon application, the following simple function has been used:

$$AC(g) = 1-(g / M)^{[(1/p)-1]};$$

[3] The term "polarizaaon" here refers to the trend for new acavities to locate where existing activities are concentrated.

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where: AC(g) = probability that a given triangle Weber problem involving a greater distance g will not be rejected because of the distance deterrence effect;

p =probability of rejecting a random triangle Weber problem because the greater distance between the three reference points is too great (when "greater distances" are uniformly distributed between 0 and M); p takes on values
between 0 and 1;

M = maximum distance between any two points in the considered space; g = greater distance between the reference points of a given triangle Weber problem.

The acceptance procedure is such that a given triangle Weber problem involving a greater distance g is rejected when AC(g) is smaller than a random value uniformly distributed
between 0 and 1.

The above procedure is not the only one that can be imagined. Traditional gravity
models and entropy maximization can also be used. For instance, the traditional negative exponential form of the gravity model suggests the following AC(g) function:

$$AC(g) = e^{[-sg]}$$

where: s = a parameter directly linked to p through the following equation:

$$p = [1 - AC(M)] / (sM).$$

Distance-based acceptance functions have a clear impact on the diffusion process of locations. The higher the distance deterrence effect, the more slowly new locations will move away from the existing locational pattern. The general impact of p is illustrated by Figure 3.

The distance deterrence effect may take into account many aspects of locational evolutions. One can even imagine a non-Weberian topodynamic model based essentially on this effect. However, such a simplified model comprises major limitations. It restricts new locations to existing location points; it cannot deal with effects linked to the existence of borders or with repulsion effects; finally, it reduces the two-dimensional locational space to a one-dimensional reality by perceiving it through a single variable: location-to-location distance.

Characterizing Location Patterns

At any point in an iterative location process, location schemes can be characterized by
the ten following indices which play an important role in the approach :

1- The parameter L of the "rank-size rule" corresponding to the distribution obtained from the ranking of the centers of activity concentration[4] A higher value of L corresponds to a center hierarchy dominated by a limited number of large centers.

[4] For the purpose of the application to the Cameroon case, the traditional form (suggested by Zip of the rank-size rule has been used

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2- The ratio Z between the size of the second rank center and the size of the first rank center.

3- The distance H between the center of gravity[5] location at time t and the corresponding location at time (t+1).

4- The angle T of gravity center shifting. This angle is measured from the north to the east.

5- The distance G between the gravity center and the geometric center of the considered space (the geometric center coincides with the center of gravity of locations when activities are uniformly distributed through space).

6- A deconcentration index Y related to the geometric center. This index takes on values between 0 and 1; the more activities are on the average located far from the geometric center, the closer to 1 the value of Y is. This deconcentration index is defined as follows:

$$Y = \left(\sum D_h\right) / \left(D_{\max} \cdot n\right);$$

where D_h = distance between activity h and the geometric center;
$D_{\max}$ = maximum distance between the geometric center and any location in the considered space;
n = number of activities.

7- A concentration index C taking on values between 0 and 1; the value 1 corresponds to the case where all activities are concentrated in a single location. Index C is defined as follows :

$$C = \sum C_k / K;$$

where $C_k = \sum \left| P_j - PA_j \right| / B_k;$

where $A_j = S_j / S;$

$$B_k = \left[2P\left(S_k - 1\right)\right] / S_k;$$

$$S_k = SJ_k / R;$$

where j = suffix referring to a subdivision in a given grid k;

K = number of grids for which an index C_k calculated; six different grids have been used: the first one being composed of 4 subdivisions; the second one of 9; the third one of 25; the fourth one of 121, the fifth one of 529 and the last one of as many subdivisions as there were points in the studied discrete space; k = suffix referring to a given grid covering the studied space;

J_k = number of subdivisions inside grid k;

P = total number of activities (or total population);

P_j = number of activities (or population) inside subdivision j ;

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[5] The gravity center is given by $\left[\left(\Sigma x_i\right) / n, \left(\Sigma y_i\right) / n\right]$ where (x_i, y_i) are the rectangular coordinates of point i.

R = area of the rectangular grid necessary for covering the studied space; S = total area of the studied space;
S_j= area of subdivision j.

The numerator of index C_k represents the sum of the differences between the observed distribution of activities and a theoretical uniform distribution of the same activities. The denominator represents the sum of the differences between a distribution in which all the observed activities are grouped together in a single location and a theoretical uniform distribution of the same activities. Index C_k and index C take on values between 0 and 1, the index being equal to 1 when all activities are grouped together in a single location.

8- A scattering index E varying between 0 and 1, value 1 corresponding to the case where all activities are grouped together in only two locations separated by the maximum distance possible in the studied space. Index E is defined as follows:

$$E = \left[\sum \left(P_i \cdot d_i\right)\right] / \left(P \cdot d_{\max}\right);$$

where P = total number of activities (or total population);
P_i = number of activities at location i ;
d_i = distance from location i to the nearest activity location;
d_{max} = maximum possible distance between two locations in the studied space.

The numerator constitutes a measure of distances between activity locations. The denominator represents a similar measure for a distribution where all activities are grouped together in two concentrations at both extremities of the studied space.

9- An index M that reflects the distribution of activities (or population) between large, medium and small activity centers; a positive variation of M indicates that, globally, the variation in the distribution of activities between large, medium and small centers favors larger centers; the opposite prevails when a negative variation of M is observed.
Index M is defined as follows:

$$M = \left(\sum \mathrm{Pr}_i\right) / V^*$$

where $\mathrm{Pr}_i = P_i / P$ = proportion of total activities located at i ;

$$V^*=V(V+1)/2$$

V = number of centers in the considered space.

It is interesting to compare the evolution of M and of the index L of the rank-size rule. Despite that both indices are generally expected to vary in the same direction, an increase of L may come with a decrease in M and vice versa.

10- A dispersal index Q defined as follows:

$$Q = \left(2 / n\right) \cdot \sum z_i \cdot \sqrt{\left(n / S\right)};$$

where $z_i = \left(\sqrt{2}\right) / 4$, if more than one activity is located in center i;

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and z_i = distance to the nearest center, if there is only one activity at location i. An increase in Q indicates an increase in the relative dispersal of activities in the considered space. It may be noticed that the index is such that when many activities are grouped together in a given center, the distance to the nearest-neighbour is assumed to be a positive constant.

These different indices can describe the initial scheme of location of reference points as well as the scheme obtained at the end of a simulation. We shall use r, g, i, and I for indices of the initial scheme whereas R, G, I, and L will denote the indices of the final scheme.

Estimating the Characteristic Parameters

The objective of topodynamic analysis consists in estimating the characteristics of an analogous Weberian process that can reproduce as precisely as possible an observed locational evolution. In order to do so, a first step is to estimate, with the help of many theoretical simulations based on the observed initial locational pattern and involving n iterations, the following functions:

$$C = \beta_1 + \beta_2 c + \beta_3 g + \beta_4 I + \beta_5 \ln\left[p/\left(1-p\right)\right] + \beta_6 \ln\left[a/\left(1-a\right)\right] + \beta_7 \ln\left[i/\left(1-i\right)\right];$$

$$G = \beta'_1 + \beta'_2 c + \beta'_3 g + \beta'_4 I + \beta'_5 \ln\left[p/\left(1-p\right)\right] + \beta'_6 \ln\left[a/\left(1-a\right)\right] + \beta'_7 \ln\left[i/\left(1-i\right)\right];$$

$$L = \beta''_1 + \beta''_2 c + \beta''_3 g + \beta''_4 I + \beta''_5 \ln\left[p/\left(1-p\right)\right] + \beta''_6 \ln\left[a/\left(1-a\right)\right] + \beta''_7 \ln\left[i/\left(1-i\right)\right].$$

The logarithmic form of these functions is dictated by the necessity to ensure that the obtained values of p, a and I do not exceed 1 or be smaller than 0.
Reliable regressions have been obtained for C and for G from 24 different scenarios involving 917 iterations[6] based on the locational pattern observed in Cameroon in 1967; unfortunately, the obtained regression for L and variable p had to be dismissed. The 2- equation with 2-unknown system that resulted is the following:

$$C = 0.502823 - 0.0007377 \ln\left[a/\left(1-a\right)\right] + 0.0082559 \ln\left[i/\left(1-i\right)\right];$$

(111.94) (-0.76) (15.95)

$$G = 29.404018 + 0.2878336 \ln\left[a/\left(1-a\right)\right] + 0.771807 \ln\left[i\left(1-i\right)\right].$$

(79.29)(3.59) (18.06)

All the above coefficients are statistically significant (the Student's t test ratios are indicated under the coefficients). Considering the fact that these results correspond to regularities noticed in stochastic processes, the obtained R2 values are remarkably high: they are the following: for

[6] The value of n is determined exogeneously and corresponds here to the increase in the urban population of Cameroon between 1967 and 1976 (during that period, the Cameroon total urban population augmented by 917 000 inhabitants).

equation C, 0.9240; and for equation G, 0.9417. The values of c, g and I being given, their effect is implicitly included in the constant.

Given the observed values of C and G for 1976 in Cameroon, the preceding equations allow for estimating the following values:

a= 90% (or more precisely 0.84638);
i=100% (or more precisely 0.99998).

Further simulations led to fix the value of p at 19%.

Calculating the Adjustments

The estimation of the p, a and i values which best fit to the observed evolution leads to calculating an "average scenario" corresponding to the spatial distribution of activities (or population) that should emerge "on the average" when 917 iterations of a weberian dynamic process characterized by the given p, a and i values take place starting from the initial pattern observed in Cameroon in 1967. This can be easily obtained by computing for each point of the studied space the average population obtained during ten different runs of the model.

Comparing these "average" populations to those observed in 1976 leads to draw maps of both negative and positive adjustments. In the case of Cameroon, the map of the populations underestimated by the "average scenario" has been most interesting (see Figure 4). It revealed a large number of centers whose growth cannot be fully explained by endogeneous factors. In fact, these centers appeared to correspond essentially to capitals (national and regional), harbours or crossroads.

Forecasting

Since the model includes no growth rates, it is not really "temporal"; it is rather "dynamic" in that sense that it simulates the evolution of urban population distributions when the total urban population increases and/or is reallocated through space. In fact the timing of the predicted evolution is a matter of exogeneous interpretation. If we may venture a medical comparison, the model pretends to predict the evolution of the disease, but not the date of cure or death... This being said, for presentation purposes, we shall date our results on the basis of exogeneous projections of total urban population.

The projections obtained from the model allows for calculating an average scenario taking account of the adjustments (this scenario is illustrated in Figure 5). The coherence of the obtained results must be stressed. Tables 1 and 2 describe the evolution of the various descriptive indices corresponding to the average projection. Values for 1967 and 1976 correspond to observations. Values between brackets concern cities registered in the 1967 census; other values take into account cities registered in the 1976 census.

It has to be noticed that, in the case of 9 of the 10 indices, the projected evolution coincides, both in terms of direction and magnitude of change, with the evolution observed between 1967 and 1976.

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The only exception concerns index Y of central deconcentration which slowly decreases between 1976 and 2016 whereas it slightly increases between 1967 and 1976. The coherence of projected and observed evolutions is remarkable considering the fact that the descriptive indices used characterize the global structuring of the urban reality in Cameroon, and not isolated parts of that reality; in other words, these indices are sensitive to any change occurring anywhere in the system.

It must also be noticed that, despite some radical change in the ranking of cities, the obtained projections do not generally contradict the observed relative values of cities growth rates. Generally, if city A has experienced a growth superior to city B's between 1967 and 1976, the projected evolution forecasts city A growing faster than city B between 1976 and 2016, even if city A and city B's growth rates significantly vary through time.

Finally, it must be noticed that the results obtained in the case of Cameroon were converging: they indicated that the ranking of cities tended to stabilize in the long run. This seems to indicate that the topodynamic analysis may reveal an ideal converging image of the simulated system which should prevail in the long run if the very dynamics of the system remain unchanged. This long-run image of the system may be very useful to understand the basic trends of urban evolution.

Conclusions

The topodynamic approach appears to provide very coherent results in a very challenging context where data are extremely scarce and space is a major variable. As far as the Cameroon application is concerned, its results are convincing, realistic and clearsighted. It is designed to take into account diffusion processes, polarization phenomena and center-periphery equilibriums. Moreover, it is not limited to existing locations; it can simulate evolutions involving the appearance of new population centers (as in the conquest of the West). Finally, it seems to provide long-run converging images which can be of a great interest in spatial analysis.

Essentially based on space-economics, the topodynamic approach could become a very useful complement to traditional demographic analysis, both in developing countries and the industrialized world. It may be the harbinger of new empirical applications of the famous Weber problem.