Analysis of Statistical Methods for Estimating Solar Radiation

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Abstract

Daily global radiation is not only required for a large number of ecological, physiological and agro-meteorological models but also is required for the design and evaluation of solar energy systems. Global radiation data is quite frequently not available for all locations. For such locations daily global radiation can be estimated statistically using continuous series of other measurable meteorological parameters. This paper considers four statistical methods for estimating the daily global radiation: Angstrom-Prescott, Coulsen, Hargreaves and Supit–van Kapel method. The empirical coefficients that characterize these methods were determined using the least squares method for the two locations near Belgrade in Serbia. The results showed that calculated empirical coefficients are similar to values that were previously determined in similar researches for neighboring countries. Further analysis verified accuracy and applicability of four empirical relations, where Ångstrom-Prescott and Supit-Van Kappel methods proved to be by far the most reliable in the assessment of daily global radiation.

Key words: Solar radiation, meteorology, Angstrom-Prescott method, empirical coefficients, Serbia, Belgrade

Introduction

For stations where the global radiation is not measured or is partly missing there are several techniques for estimating and modeling global radiation. All these models can be divided into two groups. The first group is based on the astrophysical properties of the Earth, atmospheric physics and geometry of location for which global radiation needs to be estimated (Paulescu, et al., 2013). Such modeling is called prognostic or physical modeling, where it is common to model seperately shortwave and longwave solar radiation. The second group includes those models which are based on statistical data or data obtained from satellite observations. This approach is called statistical modeling, where the most commonly used method is based on empirical relations and requires the development of equations for estimating solar radiation according to the commonly measured meteorological variables. This type of modeling is known as statistical modeling of global radiation. The most accessible and measured data are used as input data for estimating such as sunshine duration, air temperature, air temperature combined with cloudiness and air temperature combined with total daily rainfall (Trnka, et al., 2005).

Researches on the assessment of daily global radiation in this region were carried out in the previous period, but none of these studies included all four referred methods for the area of Belgrade. The best known are studies that Supit and van Kappel (1998) performed for the neighboring countries of Croatia and Slovenia, Katic et al. (1979) for Vojvodina and Paltineanu and Mihailescu (2002) for neighboring Romania.

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Methods and data

Methods

Ångstrom proposed model for assesment of daily global radiation that was later altered and improved by Prescott. According to this method daily global radiation G_d is dependent on daily solar radiation that reaches the top of the atmosphere G_a and the relative sunshine duration S_r:

$$G_d = G_a \left(a_a + b_a S_r \right)$$

where a_a and b_a are empirical coefficients for a given location (Sabziparvar, et al., 2013). Empirical coefficient a_a has physical meaning and represents the ratio of solar radiation that reaches the earth's surface on a completely cloudy day and solar radiation that reaches the top of the atmosphere. The sum of coefficients $a_a + b_a$ represents same ratio of these two variables but during a completely cloudless days (Tahâş, et al., 2011). The values of these coefficients are dependent on geographical location of station. Ångstrom-Prescott method is based on the linear relationship between variables.

Atmospheric phenomena that most affect the radiation that reaches the Earth's surface are clouds. On the basis of previous studies Supit and Van Kappel (1998) a method in which dependence is presented in the form of:

$$G_d = G_a \left[a_s \sqrt{(T_{max} - T_{min})} + b_s \sqrt{1 - \frac{n}{8}} \right] + c_s$$

where n represents daily cloudiness in eights, T_{max} and T_{min} the maximum and minimum daily temperatures and a_s, b_s, and c_s the empirical coefficients (Supit, van Kappel, 1998).

Coulson method gives the empirical relationship between the daily global radiation G_d and daily cloudiness n via:

$$G_d = G_{d0} \left[1 + (1 - \xi) n \right]$$

where introduced labels have the following meanings, G_{do} represents daily global radiation that reaches the earth's surface on cloudless day, while ξ represents constant determined by empirical means (Mihailović, 1988).

Hargreaves et al.(1985) proposed simple method for estimating daily global radiation based on daily amplitude of air temperature:

$$G_d = G_a a_H \sqrt{(T_{max} - T_{min})} + b_h$$

where a_h and b_h represent empirical constants (Hargreaves, et al., 1985).

Description of location and data sets

Ångstrom- Prescott, Supit-Van Kappel, Coulson and Hargreaves methods have been tested for two locations near Belgrade, Zeleno Brdo (44°47 N and 20°52 E, 242 m) and Pančevo (44°50' N and 20°40' E, 76 m). Data sets of measured global radiation and sunshine duration were required for testing Ångstrom-Prescott method. For verification of Supit-Van Kappel method data sets of measured global radiation, maximum and minimum daily temperatures and cloud cover were required. In Coulson equation daily global radiation that reaches the earth's surface on cloudless day had been determined by statistical analysis as product of the mean annual value of transmitance on cloudless day and daily solar radiation that reaches the top of the atmosphere. In addition to measured global radiation data sets of cloudiness were required for this method. Data sets of measured global radiation and daily temperature amplitude were required for analysis of Hargraves method. Ground measured global radiation had been taken from two meteorological stations at Zeleno Brdo and Pančevo, while data sets of daily cloudiness, sunshine duration, maximum and minimum daily temperatures have been taken from climatological yearbook of RHMS (Republic Hydrometeorological Service of Serbia) (http://hidmet.gov.rs/).

Calculation of empirical coefficients

Empirical coeficients for Ångstrom- Prescott, Supit-Van Kappel, Coulson and Hargreaves equations were calculated using the least squares method. In general case expressions for the coefficients of linear function are obtained by applying the least squares method in the form of:

$$b = \frac{\sum x \sum y - n \sum xy}{\left(\sum x \right)^2 - n \sum x^2}$$

$$a = \frac{\sum x \sum xy - \sum x^2 \sum y}{\left(\sum x \right)^2 - n \sum x^2}.$$

where x and y represent the calculated and measured daily global radiation, while n represents the number of measurements (days) (Irwan, et al., 2012).

Root mean square error (*RMSE*) in (kJ/cm^2) had been calculated in order to verify obtained results as well as the relative error in σ_{Gd} (%):

$$RMSE = \sqrt{\frac{\sum \left(G_d - G_i \right)^2}{n}}$$

$$\sigma_{G_d} = \frac{\sum \dfrac{G_d - G_i}{G_o}}{n}$$

where G_d and G_i represent the observed and estimated daily global radiation and n represents number of days for which daily global radiation was estimated and measured (Sabziparvar, 2007).

Results

Monthly values of empirical coefficients for both stations are shown in Table 1. As previously stated these coefficients are most dependent on the geographic location of station (Rahman, et. al 2005). To validate obtained results we compared values of calculated coefficients for three methods to values that Supit determined in his research for neighboring countries. Average values of calculated empirical coefficients for Ångstrom- Prescott method are approximate to the values that Supit obtained in his research for adjacent land Croatia (Zagreb, $a_a = 0.22$ and

$b_a = {}_{0.56}$) (Supit, van Kappel, 1998). A similar thing applies to the calculated values of empirical coefficients for Supit-Van Kappel method. They are approximate to values of empirical coefficients that Supit determined on the basis of long-time series of measured global radiation for adjacent land Croatia (Split, $a_s = 0.11$, $b_s = 0.36$ and Zagreb, $a_s = 0.10$, $b_s = 0.32$) (Supit, van Kappel, 1998). It must be emphasized that calculated values of empirical coefficients for Coulson method fit almost perfectly with value that Katic and associates determined in their research for Novi Sad in 1978 (Novi Sad, $\xi = 1.89$) (Mihailović, 1988). As with the first two mentioned methods average values of calculated empirical coefficients correspond to values that Supit determined for neighboring country Slovenia (Ljubljana, $a_h = 0.18$ and Portorož, $b_h = 0.22$) (Supit, van Kappel, 1998).

Ability of considered methods to truly reproduce daily global radiation in area around Belgrade and the validity of obtained results can be seen in Figure 1 where is shown correlation between the calculated and observed daily global radiation.

Table 1. *Calculated empirical coefficients for Ångstrom- Prescott, Supit-Van Kappel, Coulson and Hargreaves methods*

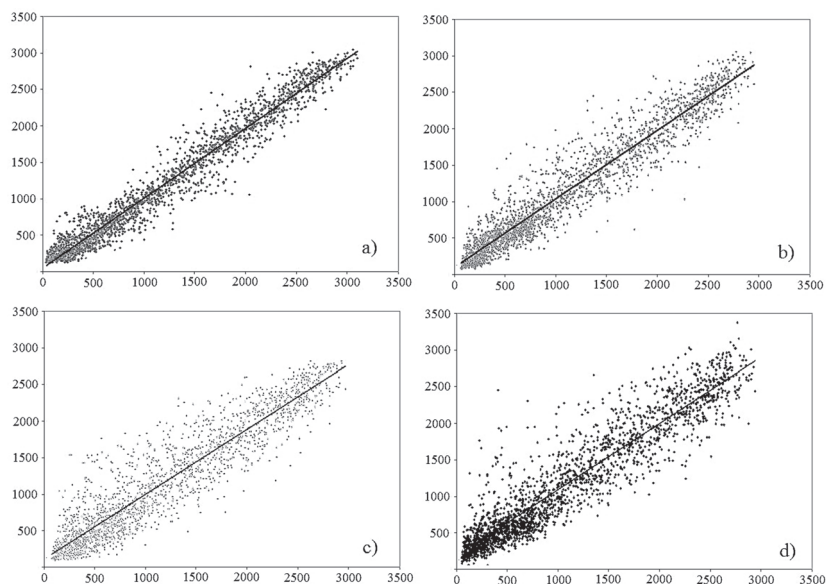| Station	Month	Ångstrom-Prescott		Supit-Van Kappel			Coulson	Hargreaves	
		a_a	b_a	a_s	b_s	c_s	ξ	a_h	b_h
Pančevo	January	0.17	0.50	0.07	0.35	0.00	1.83	0.17	-0.26
	February	0.28	0.37	0.13	0.15	0.00	1.79	0.17	-0.18
	March	0.21	0.53	0.09	0.38	0.01	1.66	0.21	-1.15
	April	0.18	0.57	0.08	0.39	-0.07	1.88	0.25	-3.03
	May	0.17	0.63	0.09	0.41	0.01	1.78	0.16	0.69
	June	0.15	0.64	0.08	0.43	-0.01	1.99	0.33	-5.85
	July	0.16	0.64	0.06	0.53	0.00	2.01	0.27	-3.27
	August	0.17	0.57	0.07	0.43	-0.01	2.25	0.22	-1.65
	September	0.12	0.57	0.07	0.35	-0.04	2.13	0.21	-1.75
	October	0.16	0.42	0.07	0.28	-0.04	1.87	0.19	-1.15
	November	0.16	0.46	0.07	0.27	0.02	1.76	0.12	0.18
	December	0.14	0.46	0.07	0.33	0.01	1.81	0.16	-0.24
	Average	*0.17*	*0.54*	*0.08*	*0.38*	*-0.08*	*1.88*	*0.19*	*-0.62*
Zeleno Brdo	January	0.20	0.50	0.08	0.35	0.01	1.78	0.19	-0.35
	February	0.21	0.55	0.07	0.45	-0.01	1.60	0.20	-0.56
	March	0.20	0.55	0.08	0.40	-0.01	1.53	0.21	-1.09
	April	0.21	0.52	0.09	0.36	-0.06	1.56	0.25	-2.69
	May	0.19	0.59	0.08	0.41	-0.04	1.31	0.27	-3.96
	June	0.19	0.57	0.07	0.46	-0.02	1.38	0.29	-4.63
	July	0.20	0.56	0.05	0.52	-0.01	1.42	0.22	-2.03
	August	0.20	0.56	0.09	0.34	-0.02	1.53	0.22	-2.04
	September	0.18	0.57	0.07	0.44	-0.06	1.73	0.28	-3.03
	October	0.17	0.58	0.06	0.43	-0.04	1.84	0.22	-1.23
	November	0.20	0.48	0.07	0.36	0.00	1.80	0.18	-0.32
	December	0.16	0.52	0.06	0.37	0.00	1.80	0.20	-0.45
	Average	*0.19*	*0.56*	*0.07*	*0.42*	*-0.05*	*1.61*	*0.18*	*-0.53*

Figure 1. Correlation between observed and measured daily global radiation for a)
Ångstrom- Prescott, b) Supit-Van Kappel, c) Coulson and d) Hargreaves methods

Table 2. *Root mean square errors (RMSE) and relative errors (σ)*

Station	Month	Ångstrom-Prescott		Supit-Van Kappel		Coulson		Hargreaves	
		RMSE (kJ/cm²)	σ (%)	RMSE (kJ/cm²)	σ (%)	RMSE (kJ/cm²)	σ (%)	RMSE (kJ/cm²)	σ (%)
Pančevo	January	106.07	27.85	149.85	40.22	152.21	38.35	204.63	54.70
	February	244.91	34.64	250.13	32.67	262.59	36.96	259.05	35.69
	March	334.01	33.86	416.38	46.53	418.11	53.75	469.80	57.30
	April	235.98	21.84	328.89	25.64	415.92	40.74	364.29	28.51
	May	178.67	6.33	293.55	10.85	333.50	13.52	373.31	14.27
	June	201.29	8.20	259.38	10.09	360.62	15.78	344.16	14.47
	July	200.74	9.60	271.80	12.48	448.82	32.00	467.73	28.36
	Avgust	228.58	11.14	309.82	16.84	327.09	17.38	364.94	19.57
	September	273.17	26.61	337.64	31.31	391.94	39.69	374.41	34.64
	October	245.91	35.31	257.51	35.47	319.71	36.13	269.30	39.81
	November	172.27	35.84	208.80	52.44	236.02	48.06	229.55	61.63
	December	99.82	44.21	122.35	48.41	122.10	49.82	149.81	62.12
	Average	*253.63*	*30.40*	*297.78*	*36.29*	*343.12*	*52.93*	*354.39*	*44.64*
Zeleno Brdo	January	91.68	32.69	115.99	33.75	128.74	38.56	164.13	53.91
	February	146.35	31.04	186.30	34.61	205.41	43.72	285.99	53.88
	March	166.81	22.75	230.38	29.52	273.28	40.23	332.71	41.53
	April	252.50	24.57	275.92	24.29	388.00	43.16	313.51	26.37
	May	208.07	12.03	317.92	18.55	451.51	31.53	382.72	21.96
	June	212.13	12.06	289.96	14.04	409.83	28.02	410.85	23.33
	July	191.25	11.24	323.70	15.38	382.91	24.72	516.70	30.06
	Avgust	193.79	10.52	382.83	23.01	420.32	26.69	422.77	26.13
	September	211.32	26.68	241.98	29.37	341.44	54.20	275.74	32.83
	October	123.31	24.68	195.35	32.28	234.15	47.32	251.86	40.88
	November	110.94	33.77	128.51	31.55	153.55	43.04	182.29	53.14
	December	87.31	53.05	97.51	39.54	109.36	49.81	141.88	61.20
	Average	*180.00*	*26.29*	*252.07*	*27.70*	*321.60*	*50.79*	*345.43*	*42.01*

It was necessary to determine relative (σ) and root mean square error (*RMSE*) for each month in order to quantitatively analyze obtained results. The results are shown in Table 2. All four methods estimate global radiation poorer during the cold period of the year. This applies especially to the months of November and December. On the other hand all four methods estimate daily global radiation very well in the months of June and July.

Conclusion

Analysis of statistical modeling of global solar radiation included the verification of the accuracy and applicability of four empirical relations: Ångstrom-Prescott, Supit-Van Kappel, Coulson and Hargreaves methods for estimating the daily global radiation. It was necessary to determine the coefficients that characterize this geographic area for each of the empirical relations. To determine the coefficients it was necessary to provide a continuous series of measured global radiation as well as the other measured meteorological variables: sunshine duration, daily cloudiness and daily temperature amplitude.

Ångstrom-Prescott method proved to be by far the most reliable in the assessment of daily global radiation. Supit-Van Kappel method has also achieved very satisfactory estimate of daily global radiation. Coulsen and Hargreaves method were chosen for testing because of the ease and availability of data, but on the other hand those two methods proved to be far less able to estimate daily global radiation. Evaluation of daily gloal radiation of the first two methods in the period from April to August is very good. Evaluations for all other months are to be regarded as satisfactory, except for December and November when the relative error is too large. The last two methods estimate daily global radiation well in warmer part of year, but do not estimate at satisfactory level in winter months. The most likely cause of large deviations of predicted value from measured value is the cloudiness that is most difficult to properly parameterize in these relations.

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